Answers for class prep quiz on section 4.9, Stewart's Calculus (8th ed.)

1. **Answer:** (a). Since the chain rule tells us that

$$\frac{d}{dx}(\sin 2x) = (\cos 2x)(2),$$

   $\sin 2x$ is *not* an antiderivative of $\cos 2x$.

2. **Answer:** (d). This is something of a trick question, in that each of (a), (b), and (c) is a particular antiderivative of $2\sin x\cos x$. (Try taking their derivatives, using the fact that $\sin 2x = 2\sin x\cos x$.) The point is that even though those functions look different, any one of them differs from any other by a constant:

$$\begin{aligned}\sin^2 x &= -\cos^2 x + 1,\\ -\frac{\cos 2x}{2} &= \sin^2 x - \frac{1}{2},\\ -\frac{\cos 2x}{2} &= -\cos^2 x + \frac{1}{2}.\end{aligned}$$

   The **most general** antiderivative of $2\sin x\cos x$ is any of these, *plus an arbitrary constant*, e.g., $\sin^2 x + C$.

3. **Answer:** (a). Recall that:

$$\frac{d}{dx}(x^4) = 4x^3, \qquad \frac{d}{dx}(\ln|x|) = \frac{1}{x}, \qquad \frac{d}{dx}(\sin x) = \cos x.$$

   Therefore, a particular antiderivative of x^3 is $\frac{x^4}{4}$; a particular antiderivative of x^{-1} is $\ln|x|$; and a particular antiderivative of $\cos x$ is $\sin x$. Putting that together with our standard rules, we see that the most general antiderivative of $8x^3 - 3x^{-1} + 5\cos x$ is

$$2x^4 - 3\ln|x| + 5\sin x + C.$$

4. **Answer:** (b). Given $f''(x) = 7x^3 + e^x$, the most general antiderivative of $7x^3 + e^x$ is $\frac{7x^4}{4} + e^x + C_1$, so $f'(x) = \frac{7x^4}{4} + e^x + C_1$. However, since $f'(0) = -3$,

$$-3 = \frac{7(0^4)}{4} + e^0 + C_1 = 1 + C_1,$$

so $C_1 = -4$, and $f'(x) = \frac{7x^4}{4} + e^x - 4$.

Similarly, the most general antiderivative of $f'(x) = \frac{7x^4}{4} + e^x - 4$ is $\frac{7x^5}{20} + e^x - 4x + C_2$, so $f(x) = \frac{7x^5}{20} + e^x - 4x + C_2$. However, since $f(0) = 11$,

$$11 = \frac{7(0^5)}{20} + e^0 - 4(0) + C_2 = 1 + C_2,$$

so $C_2 = 10$, and $f(x) = \frac{7x^5}{20} + e^x - 4x + 10$.